The present research shows the kinetic analysis of an ankle rehabilitator, the rehabilitation mechanism consists of a fixed platform in which there are two inverted delta robots linked a mobile platform, actuator are controlled by servomotors which provide six basic movements of ankle rehabilitation like dorsiflexion, plantar flexion, inversion, eversion, abduction and adduction, which was verified through a motion study and singularity analysis. The design of the rehabilitator is based on the establishment of a methodology that allows the development of a machine that achieves all the ergonomic, technological and quality requirements, using tools of computational mechanics that allowed generating a virtual model able to generate the required movements and supporting the mechanical stress generated. The mechanism was modeled in CAD software such as SolidWorks, with the virtual model is possible to analyze the inverse and direct kinematics determining the position and speed of the joints, for the selection of the servomotors the dynamic analysis was made, obtaining the accelerations, forces and torques using the MSC Adams software. For the analysis of deformations, normal stresses and shear forces, the CAE software such as ANSYS was used, which works on the basis of the finite element method, with its Workbench platform and its structural analysis module. Finally, the machine was built using 3D printing and performance tests were carried out.
1. Robotic rehabilitation

1.1. Introduction

The Lower limbs injuries and the ankle injuries are very frequent so they require highly qualified professionals for the rehabilitation process, making it necessary to develop machines that help the rehabilitation processes quickly and without risk to cause harm to the patient.

By designing a system of inverted delta robots we can solve problems such as: efficiency in ankle rehabilitation, prolonged ankle rehabilitation without fatigue of the rehabilitator, suppling the demand of people that need of rehabilitation of the ankle and giving solution to athletes who usually suffer the ankle injuries and they need rehabilitation therapies.

In the rehabilitation of the ankle, unique movements are generated, these movements describe the rehabilitation process of the ankle with mobility ranges, and these movements are described below:

Dorsiflexion, plantar flexion, inversion, eversion, abduction / external rotation, adduction / internal rotation, these rehabilitation movements are shown in Figure 1 [13].

![Figure 1. Ankle movements for rehabilitation process.](image)

Figure 1 shows the movements of ankle rehabilitation, these movements have permissible ranges, Table 1 shows the movements with the necessary ranges for a correct rehabilitation process without causing harm to the patient, and the design of the robot for rehabilitation is based on the permissible ranges shown in Table1.
A parallel robot is composed of a final effector with \( n \) degrees of freedom and a fixed base, joined together by at least two independent kinematic chains. The robot is driven by \( n \) simple actuators \cite{9}. Currently the development and applications of delta robots has been very different, due to its speed and precision being widely used in selection processes, 3D printers, etc.

1.2. Design

The design methodology includes the design procedures to get the proposed objective \cite{5}, \cite{6}, \cite{11} and \cite{12}. Figure 3 shows the following stages to get a robotic rehabilitator prototype: preliminary studies, conceptual design, specification design and materialization design.
Based in data from Table 1 the prototype allow to reach the movement for ankle rehabilitation.

- The dorsiflexion movement reaches 25° of inclination of the platform.
- The plantar flexion movement reaches 45.8 of inclination of the platform.
- The inversion movement reaches 19.5° of inclination of the platform.
- The eversion movement reaches 15° inclination of the platform.
- The abduction movement reaches 22° of inclination of the platform.
The adduction movement reaches 30.06 ° of inclination of the platform. Figure 4 shows this position.

**Cinematic Analysis**

The resolution of the direct kinematic problem allows knowing which is the position \((x, y, \text{and } z)\) and orientation \((\theta, \phi \text{ and } \varphi)\) adopted by the effector of the robot when each of the variables that limit the orientation of its joints takes determined values, so that to solve a direct kinematic problem it will be given by the relations of position and speed:

\[
x = f_x(q_1, q_2, q_3, q_4, q_5, q_6) \quad y = f_y(q_1, q_2, q_3, q_4, q_5, q_6) \quad \theta = f_\theta(q_1, q_2, q_3, q_4, q_5, q_6) \quad \phi = f_\phi(q_1, q_2, q_3, q_4, q_5, q_6) \quad \varphi = f_\varphi(q_1, q_2, q_3, q_4, q_5, q_6)
\]

\[
x = \sum_{i=1}^{n} \frac{\partial f_x}{\partial q_i} \dot{q}_i \quad y = \sum_{i=1}^{n} \frac{\partial f_y}{\partial q_i} \dot{q}_i \quad \theta = \sum_{i=1}^{n} \frac{\partial f_\theta}{\partial q_i} \dot{q}_i \quad \phi = \sum_{i=1}^{n} \frac{\partial f_\phi}{\partial q_i} \dot{q}_i \quad \varphi = \sum_{i=1}^{n} \frac{\partial f_\varphi}{\partial q_i} \dot{q}_i
\]

Based on the equations shown, Figure 5 shows the complete kinematic analysis of all the actuators based on the position of the final effector generated by MSC Adams.
The kinematic analysis of the delta parallel robot allows demonstrating all the ranges of movement required in the ankle rehabilitation, besides the speeds and accelerations are within the permissible range provided by specialists in ankle rehabilitation.

**Dynamic Analysis**

The dynamic studies the forces that act on an object and the movement generated in it. (Barrientos, 2007). For the calculation of the dynamic model of a system of articulated links, the Lagrangian formulation established by the equations shown below will be used.

\[
L = E_c - E_p \quad (13)
\]

\[
\tau_i = \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} \quad (14)
\]

Where:
- \( L \) is the Lagrangian function
- \( E_c \) is the kinetic energy
- \( E_p \) is the potential energy
- \( q_i \) corresponds to the coordinates of the joints
- \( \tau_i \) is the torque applied to the degree of freedom \( q_i \)

The dynamic simulation of the robot allows knowing what type of actuator is required to achieve the movements in the rehabilitation process. In the simulation, a static load of 100 N is considered as a reference value. Figure 6 shows the Torque-Speed curves of the different actuators of the robot, the curves allow to select the motors for the control.
Mechanical strength analysis

Finite element method (MEF) is a numerical method for the approach of solutions of complicated partial differential equations used in various engineering problems. Figure 7 shows the deformation due to the load of 100N, the simulation was generated in the ANSYS software where the deformation ranges can be verified due to the load.

2. Results

Results of kinematic analysis:

- The results obtained from the kinematic analysis are as follows: The maximum angular velocity of the actuator_1 is 63.15 deg. / s, the maximum angular velocity of the actuator_2 is 58.58 deg. / s, the maximum angular velocity of the actuator_3 is 53.46 deg. / s, depending on the position of the effector_1 which is 407.342 mm.
- The maximum speed of the actuator_4 is 34.0488 mm. / s, the maximum speed of the actuator_6 is 23.7753 mm. / s depending on the position of the effector_2 which is 174.029 mm. the maximum speed of the actuator_5 is
24.5336 deg. / s, depending on the position of the effector_2 which is 181.4925 mm.

**Dynamic analysis results:**
- The results of the dynamic analysis are described below: The maximum torque reached by motor_01 is 1559.7509 N mm. The maximum torque reached by the motor_02 is 2633.2061 N mm. The maximum torque reached by the motor_03 is 4689.27 N mm. The maximum torque reached by the motor_04 is 3595.1272 N mm. The maximum torque reached by the motor_05 is 2594.1011 N mm. These values are given as a function of time, the maximum value being that of motor_03, this value helps us to select the actuators for the robot.

**Results of resistance analysis:**
- The analysis of mechanical resistance of the delta robot, where the maximum deformation is 0.16177 mm with a load of 100N and the composition of Aluminum with density = 2770 kg / m3 shows that the structure will work properly without suffering damage and the equivalent effort von Mises: 21,912Mpa MAX - 1,2083 x 10^-8Mpa MIN.

**Tests on the prototype**
- The tests in the prototype show that the generated simulations were adequate for the design, obtaining that the prototype generates the necessary movements for the rehabilitation of the ankle.

![Figure 8. Prototype of ankle rehabilitator generated by 3d impression.](image)
References


