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DYNAMICS MODELING AND CONTROL OF A QUADROTOR SUBJECTED TO A VARIABLE LOAD

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The control law design method was applied to a quadrotor as an underactuated mechanical system (UMS). The challenges included dynamic modeling, stabilization of the underactuated system, and variable load. By definition, a quadrotor is an underactuated system since it has six degrees of freedom and only four inputs. Research on UMS is one of the most interesting topics in the robotics and control community because the traditional control strategies developed for fully actuated systems are not directly applicable to UMS. A quadrotor, like other nonlinear dynamic systems, is subject to model uncertainties which may cause instability and inaccuracy. Thus, the control of this nonlinear system is a problem for stability and tracking control considering the dynamic modeling and actuator failure. This work represents a continued development of previously published works, and the major contribution is demonstrated for assuring the effectiveness of the techniques when a quadrotor is subjected to a variable load with no modifications to the reference model. The quadrotor model is validated using MAPLE and MATLAB environment simulation.

1. Introduction

An underactuated mechanical system (UMS) is a system that has more degrees of freedom (DOF) to be controlled than the number of independently controlled actuators exerting force or torque onto the system. The modeling, formulation, and control theory of UMS have been studied to a great extent in the last decade; specifically, the control of an UMS is including in practical applications of aerial and underwater vehicles. In terms of the number of actuators, mechanical systems are fully actuated or underactuated. A mechanical system can be designed and built as an UMS. Furthermore, even a system designed as a fully actuated system could become underactuated due to an actuator failure or to a moving load. Despite the importance of the UMS control problem in real-world applications, most available control strategies are only applicable to fully actuated mechanical systems.

The Direct Lyapunov Approach (DLA) chosen to be implemented as strategy method for this work, was first presented in [1] and was applied to the stabilization of a class of systems characterized by dynamic equations where the nonlinearities depended on only one generalized coordinate and generalized velocity. The applications consisted of the inverted pendulum cart and the inertia wheel pendulum. Applying DLA to more complicated systems showed that certain matrices used in the formulation did not return to the original form after equilibrium was reached, a difficulty that altered the system dynamics during subsequent disturbances. This difficulty was addressed in [2] where the formulation was changed so that a matrix associated with the kinetic energy was made to return to the same form as equilibrium was approached.

The resulting formulation was successfully applied to the stabilization of the ball and beam system. The formulation was such that the matrix $\mathbf{K}_D$ essentially stayed constant during the stabilization period.

Further testing of the approach showed that the procedure used to make $\mathbf{K}_D$ return to a nominal form also had the tendency to drive the rate of change of the candidate Lyapunov function to zero and in some cases even positive and thus limiting the basin of attraction for stabilization of the system. A better formulation of the problem addressed in [2] was presented in [3] where certain parameters were introduced to preserve the sign of the candidate Lyapunov function rate of change to improve the performance, and in [4] applied for tracking control.

The Solvability of the Direct Lyapunov First Matching Condition in Terms of the Generalized Coordinates was presented in [5]. This work also relies on a matching equation solution method providing a robust method to solve the first matching condition.

The DLA method and a different technique were implemented to solve problems in agricultural applications using an Unmanned Hovercraft addressed in [6] and [7], respectively.

More recently different techniques were applied to a quadrotor to improve performance including underactuation, model uncertainty, and actuator failure as appeared in [8].

Improvements in the DLA method problem of the inverted pendulum cart as a nonlinear model of an underactuated two degree of freedom system was addressed to achieve stability from the proper shape of the potential, the positive definiteness of the $\mathbf{K}_D$ matrix, and the non-positive rate of change of the Lyapunov function in [9].

The present paper continues the development of the DLA for a more complicated system. A typical quadrotor has four actuators to navigate in a 6 DOF configuration space. The control constraints of a quadrotor complicate stabilization significantly, but also strong subsystems coupling, actuators failure, and system disturbances can compromise the quadrotor controller performance. To design a robust motion controller in this review the main control constraints are attributed to dynamic modeling, stabilization of the underactuated system, and variable load.

Figure 1. Unmanned Vehicle with Varying Load.

2. Modeling system dynamics

Over the past few decades, many sufficiently robust controls techniques have been developed for fully actuated systems but almost none of them are directly applicable to an underactuated mechanical system (UMS). For underactuated systems, some techniques for optimal, robust, and adaptive control have been developed lately. Some of these techniques rely on the derivation of the controller follows in equation (1), in many respects, that was presented in [2].

$$\frac{d}{dt}\left(\frac{\partial L(\boldsymbol{q},\dot{\boldsymbol{q}})}{\partial \dot{\boldsymbol{q}}}\right)-\frac{\partial L(\boldsymbol{q},\dot{\boldsymbol{q}})}{\partial \boldsymbol{q}}=\boldsymbol{Q}' \tag{1}$$

where the vector $\boldsymbol{q} \in \Re^{n}$ is a set of generalized coordinates for the n degrees of freedom of the mechanical system while the time derivative of q specifies the n generalized velocities. $L(\boldsymbol{q},\dot{\boldsymbol{q}}) : \Re^{2n} \rightarrow \Re$ is the Lagrangian defined as the kinetic energy minus the potential energy of the mechanical system. The vector Q' contains the constraints and applied control input forces/moments.

The mechanical system is described by the nonlinear matrix equation

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{C}_D\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{\tau} \\ 0 \end{bmatrix} \tag{2}$$

The right-hand side of eq. (2) contains the vector $\boldsymbol{\tau} \in \Re^{m}$ where $m<n$ for underactuated systems. It is assumed that the degrees of freedom are ordered so that the first m elements of the right-side vector contain the nonzero inputs. Also, in eq. (2), $\mathbf{M(q)} \in \Re^{n\times n}$ is the positive definite mass and/or inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \in \Re^{n}$ consists of centripetal and Coriolis forces and/or moments, and $\mathbf{G(q)} \in \Re^{n}$ consists of forces and/or moments stemming from gradients of conservative fields.

3. Quadrotor Dynamic Model

A quadrotor has six degrees of freedom and only four actuators. Its mechanism is modeled in a cross configuration with symmetrical arms. The main advantage of its symmetry is to allow for centralization of the payload and its control system. The system state variables are controlled using throttle (thrust), roll, pitch, and yaw that are directly related to the propellers' velocities. This system is underactuated because the external generalized forces are not able to accelerate its states in all directions [3].

The system geometry of the quadrotor together with the dynamic equations of motion are shown in figure 2.

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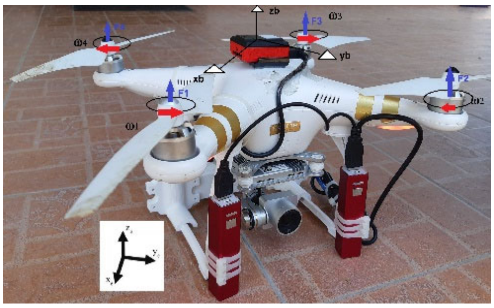

$$\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{z} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & mw & -mv \\ 0 & 0 & 0 & -mw & 0 & mu \\ 0 & 0 & 0 & mv & -mu & 0 \\ 0 & 0 & 0 & 0 & I_{zz}r & -I_{yy}q \\ 0 & 0 & 0 & -I_{zz}r & 0 & I_{xx}p \\ 0 & 0 & 0 & I_{yy}q & -I_{xx}p & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} +$$

$$\begin{bmatrix} mgsin(\theta) \\ -mgcos(\theta)\sin(\phi) \\ -mgcos(\theta)\sin(\phi) \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix}.$$

Figure 2. Quadrotor with Body and Earth Coordinate Frame, and Equations of motion.

where the quadrotor linear velocity vectors with respect to the body reference frame are (u, v, w), and the angular velocity vectors with respect to the body reference frame are (p, q, r),

Looking at figure 2, it is shown that a motor in front and one in the rear turn counterclockwise while the other pair of motors rotate clockwise. Movement occurs by increasing or reducing the rear and front engines speed [8].

The generalized coordinates of the vehicle are $q = (x, y, z, \phi, \theta, \psi)$, where (x, y, z) represent the relative position of the center of mass of the quadrotor with respect to an inertial reference, and (ϕ, θ, ψ) are the three angles of Euler representing the orientation of the vehicle.

Applying DLA to the generalized coordinates, the energy of the stabilized system is given by

$$V(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{K_D}\dot{\mathbf{q}} + \Phi(\mathbf{q}) \tag{3}$$

where

$\Phi(\mathbf{q})$: $\Re^n \rightarrow \Re$ is a potential function, and $\mathbf{K_D} \in \Re^{n\times n}$ is a symmetric, positive definite matrix defined as

$$\mathbf{K_D} = \mathbf{P(t)M(q)} \tag{4}$$

Solving equation (2) for the generalized accelerations and substituting this into equation (3) results in three matching relations mentioned in [9].

4. Control Development

The whole control algorithm is used to give the right signal to the four propellers and the six variables are controlled through the DLA method. According to the configuration of the model x and y are the unactuated axes. The set of ordinary differential equations corresponding to the three matching equations are evaluated to find $\boldsymbol{K_D}$, $\boldsymbol{K_V}$ and the potential, as mentioned in [9]. The matrix $\boldsymbol{K_D}$ remains constant to avoid complications, and $\boldsymbol{K_V}$ is found for which the solution is

$$\boldsymbol{K_v} = \sum_{i=1}^{m} \alpha_i \boldsymbol{P}_i \boldsymbol{P}_i^T \tag{5}$$

where the α_i are constants chosen so that $\boldsymbol{K_v}$ is positive semi-definite and $\boldsymbol{P_i}$ is the i^{th} column of $\boldsymbol{P}(\boldsymbol{q})$.

For the quadrotor model the inertia matrix system is a diagonal and constant, the Coriolis and centripetal matrix, and the gravitational vector are given from the mass matrix and body frame velocities and acceleration due to gravity, respectively.

5. Simulation Results of Designed Controller

The physics of the quadrotor is represented in the MAPLE file. The potential shows the correct shape (concave upward) which allow us to determine the design parameters for the $\mathbf{K_D}$ matrix, and the $\mathbf{K_V}$ matrix.

The control is designed, and the inputs are carried to a file in MATLAB where the tuning for stabilization is developed. It provides the position, velocity, and acceleration of both linear and angular quantities.

To verify the effectiveness and the application effect of the control method first, no variation of load is applied. The definitions of the physical parameters are presented in Table 1.

Table 1: Parameters of the Quadrotor Model

Parameter	Value	Unit
m	1.5	*kg*
l	0.205	*m*
g	9.8	m/s^2
I_{xx}	$8.8e^{-2}$	$kg.\ m^2$
I_{yy}	$15.2e^{-2}$	$kg.\ m^2$
I_{zz}	$23.1e^{-2}$	$kg.\ m^2$

The simulation results are shown in Figure 3 and 4.

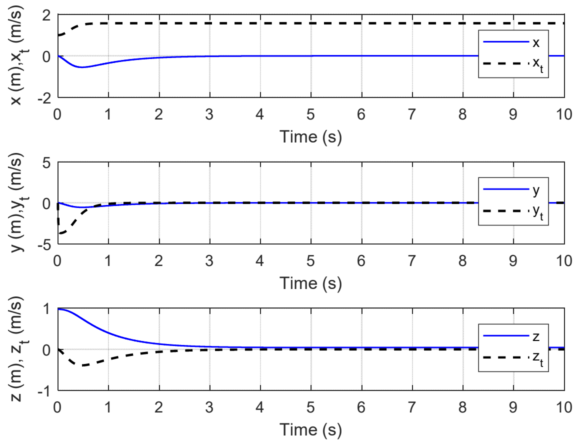

Figure 3: Quadrotor Linear Positions and Velocities

By comparing both, it can be concluded that the designed controller can effectively cancel the oscillation on the movement. In addition, the position and velocity for the linear and angular movement are converged to zero after a few seconds when input torque are implemented.

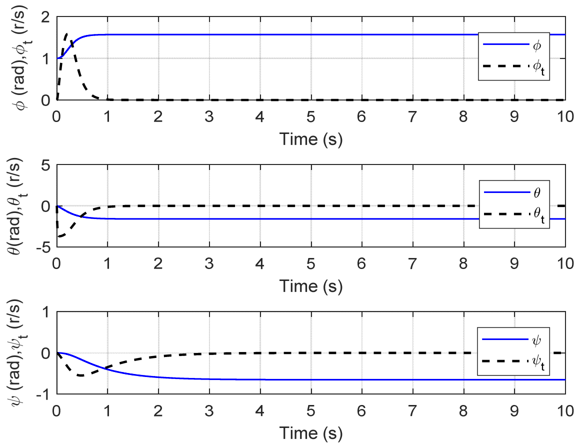

Figure 4: Quadrotor Angular Positions and Velocities

The performance of the controller outputs is shown in figure 5, where each actuator is associated with their respective direction. For instance, torque 3 performs in direction z, torque 4 performs in direction phi, torque 5 performs in direction theta, and torque 6 performs in direction psi. As it is noticed the quadrotor requires more time to stabilize in the psi direction.

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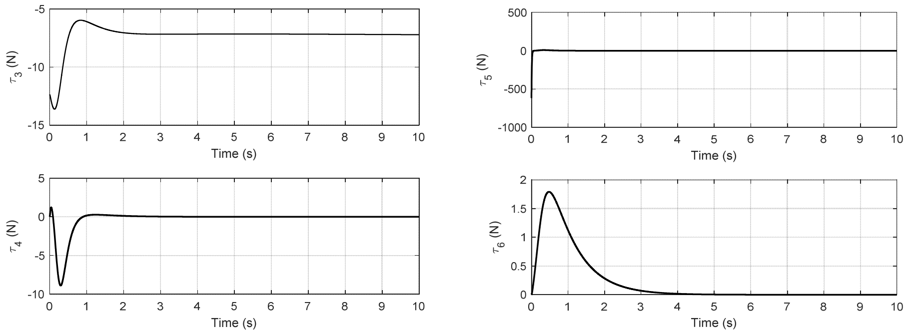

Figure 5: Control law

The Lyapunov responses is shown in figure 6, where the monotonic positive definite performance and the non-positive responses for the derivative of it is shown.

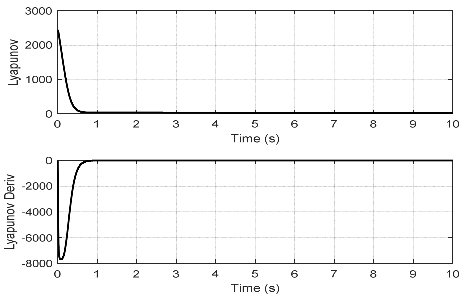

Figure 6: Lyapunov and Time Derivative History

5.1. *Responses to different loads*

The simulation routine has been carried on the quadrotor with variation of load.
In this simulation, robustness of the controller is tested by adding different loads with the reference value of 1.5 kg to the masses in all three body axes of the quadrotor, corresponding to different load disturbances. Reference values were selected to be the same as in the previous simulation. The proposed controller effectively rejects load disturbances in a range of 1.8 Kg, and the largest load 2.0 Kg perfectively.
In figure 7, the response for the controller in the z direction is shown in the presence of load variation,

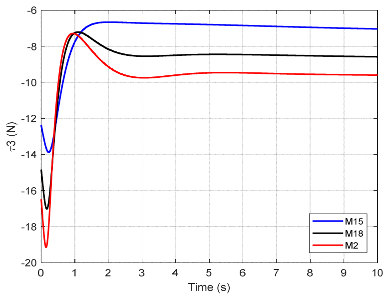

Figure 7: Control Torque in the presence of load variation, (Actuator 3 - z direction).

where control torque in the presence of load variation of 1.5 Kg, 1.8 Kg, and 2.0 Kg corresponding to M15, M18 and M2 on the figure 7, respectively.
In figure 8, the response for the controller in the phi direction is shown in the presence of load variation of 1.5 Kg, 1.8 Kg, and 2.0 Kg.

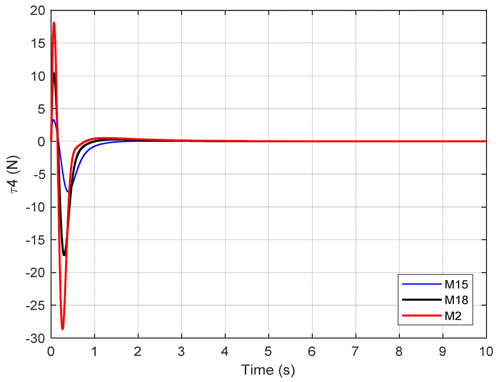

Figure 8: Control Torque in the presence of load variation, (Actuator 4 - ϕ direction).

In figure 9, the response for the controller in the theta direction is shown in the presence of load variation of 1.5 Kg, 1.8 Kg, and 2.0 Kg.

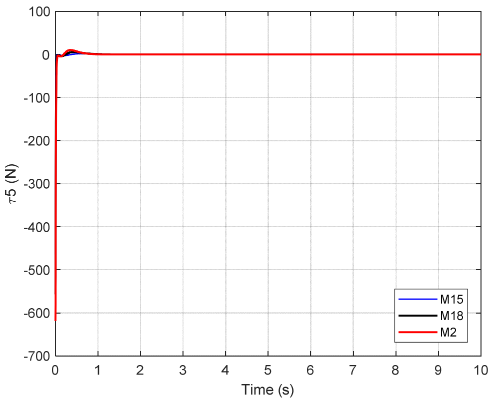

Figure 9: Control Torque in the presence of load variation, (Actuator 5 - θ direction).

In figure 10, the response for the controller in the psi direction from 4 seconds to 7 seconds as is shown in the presence of load variation of 1.5 Kg, 1.8 Kg, and 2.0 Kg. Notice that this is the latest time for a response, and more energy is required to stabilize the system with the increasing of the load to get effectiveness in the controller for this direction.

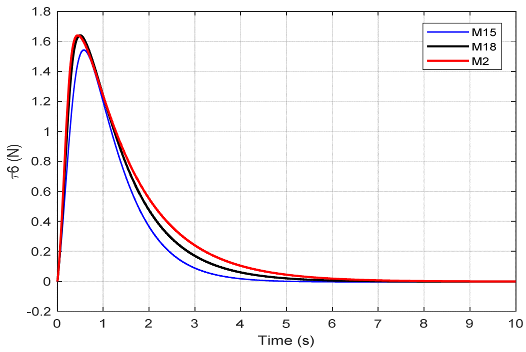

Figure 10: Control Torque in the presence of load variation, (Actuator 6 - ψ direction).

6. Conclusion

In this paper, a controller for the quadrotor is addressed with a reference load. Performance of the controller is evaluated in various load simulation scenarios.

Many different control algorithms have been proposed to control these systems; however, modifications of the system and adaptive techniques have been implemented. To this end, a dynamic model of this system was obtained, and a control algorithm was presented for the stabilization of the quadrotor.

Using simulation routines, it was shown that this designed controller could control the quadrotor motion on the different directions, also those with no actuation. This controller was designed by implementing the DLA method and considering dynamic modeling, underactuation, and variation of mass.

In this paper, the effectiveness of the controller to control the quadrotor was demonstrated.

Our future work will focus on implementing the proposed controller to enable higher loads including another degree of freedom as liquid slosh in drones.

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