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TIME DOMAIN ANALYSIS USING PID CONTROLLERS FOR UNMANNED AERIAL VEHICLES

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Abstract. The application of Unmanned Aerial vehicles in the last decade has increased rapidly. The attitude and altitude control are one of the hot topics for research due to its dynamic characteristics. The important parameters are pitch, yaw and roll which describes the 3-dimensional position of the quad-copter in space. This paper presents the design of PID controller by linearizing the non – linear equations describing the quad-copter. The time domain specifications obtained through simulation in MATLAB and SIMULINK for various altitude, pitch, roll and yaw rates describes the effectiveness of the proposed approach.

Keywords: Quad-copter, Control, Dynamic, Roll, Pitch, Yaw, Non-linear, PID.

1. Introduction

Unmanned Aerial Vehicles (UAV) is a subsystem of Micro Air Vehicles (MAV) [1]. There are various subcategories in UAV such as a cyclocopter, fixed-wing, quad-copter, flapping-wing, ducted-fan, blimp, spin copters, hexacopter, and various others. Figure 1 shows the classification of aerial vehicles where VTOL stands for vertical takeoff and landing. Cyclo-copters come under these categories. Among all UAVs, quad-copters are commonly used for several applications because of their simple structure, small volume, and low machinery accuracy [3-6]. Drones are used in aerial surveillance, rescue operation, and remote sensing. UAV with four propellers is called quad-copter which is popularly known for its high flexibility.

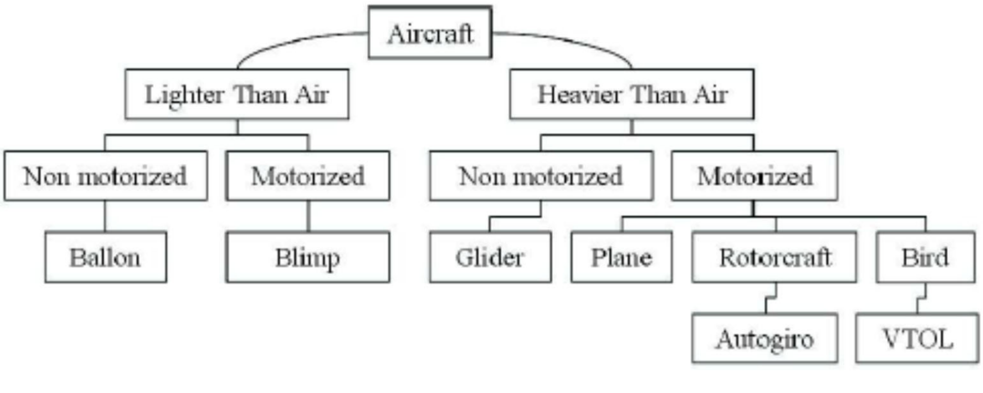

Figure 1. Classification of Aircraft based on flying principle [2].

The paper is organized as follows. The basic concepts involved in the working of quad-copter and their configurations are discussed in the next section. Section 3 is about the dynamic modeling of the quad-copter which involves the equation of motion. Section 4 will focus on the Controller block diagrams and the simulated results.

2. Working and Configuration

2.1. *Working*

The frame of the quadcopter should be symmetric [7]. Quadcopters have 4 actuators (i.e.) rotors which acts as four input forces to generate thrust and are placed with a fixed angle in between the frames [8], [9]. In drones, only the rotor speed is controlled which in turn controls the altitude and attitude. For altitude, all four rotors act at the same speed to rise to a certain level whereas for other movements like pitch, the speed of rotors in the back is increased while the speed of front rotors is decreased.

2.2. *Configuration*

The two commonly developed quadcopter designs are “+” and “x”. Figure 2 shows both the configurations of the quadcopter. X configuration is more stable compared to + configuration which is more acrobatic [10]. The different speeds of the rotors enable the quadcopter to perform roll, pitch, and yaw motion. One main advantage of using X configurations is the camera arc clearance. X configuration can have camera pointed forward without obstruction from the frame. Some drones are designed in such a way that the camera is mounted on a standoff. Availability of thrust for each movement is high in X configuration than + because of which the quadcopter is more responsive.

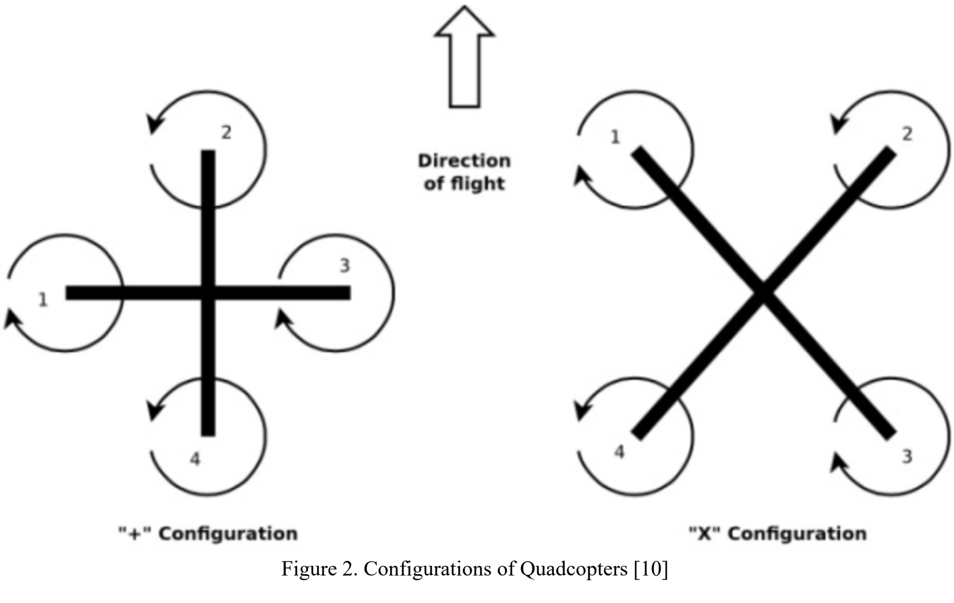

Figure 2. Configurations of Quadcopters [10]

3. Modelling and Simulation of Quadcopter

A study on performance of quadcopter was carried out using L1 controller [11]. The entire quadcopter should be represented in the form of mathematical equations which can be modelled, and results can be simulated using MATLAB and SIMULINK [12]. Important quantities that determine the stability and dynamics of the quadcopter are roll, pitch, yaw angles and its position in 3-dimensional space.

The landing problems of UAV are addressed by designing a mechanical bird like feet for landing and then analyzed dynamics of the system mathematically [13]. In this research, the entire quadcopter system could be represented in equations with 6 degrees of freedom. The most important parameters out of the 6 parameters are roll, pitch, yaw and altitude for which we have designed the controller.

3.1. *Transformation Matrix*

Transformation matrix is a matrix which is used for transformation of points from inertial to body frame of the quadcopter.

Roll: Rotational movement about the x – axis.

$$R_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & Cos\emptyset & Sin\emptyset \\ 0 & -Sin\emptyset & Cos\emptyset \end{bmatrix} \tag{1}$$

Pitch: Rotational movement about the y – axis

$$R_1 = \begin{bmatrix} Cos\theta & 0 & -Sin\theta \\ 0 & 1 & 0 \\ Sin\theta & 0 & Cos\theta \end{bmatrix} \tag{2}$$

Yaw: Rotational movement about the z – axis

$$R_2 = \begin{bmatrix} Cos\psi & Sin\psi & 0 \\ -Sin\psi & Cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Rotation matrix is the matrix multiplication of R2, R1 and R0 which defines the transformation from the inertial to the body frame.

$$R^e_b = \begin{bmatrix} C\Phi C\psi & S\Phi S\theta S\psi - C\Phi S\theta & C\Phi S\theta C\psi + S\Phi S\psi \\ C\theta S\psi & S\Phi S\theta S\psi + C\Phi C\theta & C\Phi S\theta S\psi - S\Phi S\psi \\ -S\theta & S\Phi C\theta & C\Phi C\theta \end{bmatrix} \tag{4}$$

3.2. *Newton-Euler Formalism*

Newton Euler formulation describes the dynamics of the system in terms of force and momentum acting on that system.

$$\begin{bmatrix} \mathrm{m}I_{3*3} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \omega m v \\ \omega I \omega \end{bmatrix} = \begin{bmatrix} f \\ \tau \end{bmatrix} \tag{5}$$

Here m is the mass of the quadcopter, which is 0.7 KG and V, ω are the linear and rotational velocities. f is the total linear forces acting on the quadcopter and τ is the rotational forces. I is the identity matrix with three rows and three columns.

The four motor inputs and the gravity also contribute to the net force and rotational force of the quadcopter which could be represented in the equation as follows. R_b^e is the rotational matrix obtained in equation 4

$$\begin{bmatrix} f \\ \tau \end{bmatrix} = \begin{bmatrix} R_b^e & 0 \\ 0 & I_{3*3} \end{bmatrix} [U] - G \tag{6}$$

Where,

$$[\mathrm{U}] = \begin{bmatrix} F1 + F2 + F3 + F4 \\ F4 - F2 \\ F3 - F1 \\ -F1 - F2 + F3 + F4 \end{bmatrix}$$

$$R_b^e = \begin{bmatrix} C\Phi C\psi & S\Phi S\theta S\psi - C\Phi S\theta & C\Phi S\theta C\psi + S\Phi S\psi \\ C\theta S\psi & S\Phi S\theta S\psi + C\Phi C\theta & C\Phi S\theta S\psi - S\Phi S\psi \\ -S\theta & S\Phi C\theta & C\Phi C\theta \end{bmatrix}$$

F1, F2, F3, F4 are the four thrust input which will be given as an input to the four rotors of the quadcopter. First input has four inputs acting at the same time which is for hovering of quadcopter to a certain altitude in space.

Comparing equations 4 and 6, the equation could be rewritten as follows.

$$\begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} = \frac{1}{\begin{bmatrix} \mathrm{m}I_{3*3} & 0 \\ 0 & I \end{bmatrix}} \left(\begin{bmatrix} R_b^e & 0 \\ 0 & I3*3 \end{bmatrix} [U] - G - \begin{bmatrix} \omega m v \\ \omega I \omega \end{bmatrix} \right. \tag{7}$$

The above equation could be reduced to a set of non – linear equations with 4 degrees of freedom.

$$\ddot{Z} = -g + (\cos\Phi\cos\theta)\frac{U1}{m} \tag{8}$$

$$\ddot{\emptyset} = \dot{\theta}\dot{\psi}\left(\frac{\dot{I}y-\dot{I}z}{Ix}\right) - \frac{Jx}{Ix}\dot{\theta}\,\Omega - \frac{l}{Ix}U2 \tag{9}$$

$$\ddot{\theta} = \dot{\emptyset}\dot{\psi}\left(\frac{\dot{I}z-\dot{I}x}{Iy}\right) - \frac{Jx}{Iy}\dot{\emptyset}\,\Omega - \frac{l}{Iy}U3 \tag{10}$$

$$\ddot{\psi} = \dot{\emptyset}\dot{\theta}\left(\frac{\dot{I}\dot{x}-\dot{I}y}{Iz}\right) + \frac{1}{Iz}U4 \tag{11}$$

Here Z represents the Z-axis which is basically the altitude. The above equations are non-linear and hence it is difficult to control the system with controllers like Proportional Integral Derivative controller (PID). The above equations could the further simplified by linearizing the non – linear equations. They are linearized by assuming that the quadcopter is flying in a nominal equilibrium condition. Nominal equilibrium is a steady state trajectory where $\dot{\emptyset}$ & $\dot{\theta}$ equals to zero.

$$\ddot{Z} = \frac{U1}{m}(-g+1) \tag{12}$$

$$\ddot{\emptyset} = \frac{l}{Ix}U2 \tag{13}$$

$$\ddot{\theta} = \frac{l}{Iy}U3 \tag{14}$$

$$\ddot{\psi} = \frac{1}{Iz}U4 \tag{15}$$

3.3. *Motors and Propellers*

DJI Maveric drone is taken as a reference for the dimensions. The mass of the drone is 0.743 Kg and the frontal area to 0.0197 m^2. The diagonal length of the frames of the proposed system is to be 335mm. Maveric Pro uses a 3S Lithium Polymer battery which is commonly well known for its high discharge rate. Similarly, the same 3 cell lithium battery is deployed. The deployed battery model is Turnigy 3000mAh 3S 20C Li-Po Pack w/XT-60. As the model's name itself suggests, the maximum capacity is 3000mah with 11.1V as nominal voltage. The power consumption study for motors was carried out when they are attached with a landing foot which could be used for automatic landing and also ultrasonic sensor is used for this purpose [8].

Brushless DC motor is the motor that is commonly used in drones because it is more powerful compared to a normal DC motor. Further, a fixed stator is wound by coils and a rotor with electromagnet. DYS MR2205 – 2750 is the motor chosen which has an operating voltage of which matches the battery specification. Here the diameter of the stator is 22mm and 5mm length. The motor constant is 2750 VK which is the number of RPM per volt. Using the datasheet of DYS MR2205 – 2750 and performance datasheet from ATC propeller manufacturer, the correlation between voltage and RPM, coefficient of thrust and RPM, torque and coefficient of thrust is obtained by plotting the important parameters from the datasheet and finding the best fit equations which are listed below.

$$\text{RPM} = (-62.0)\, \boldsymbol{V^2} + (2750.1)\text{ V} \tag{16}$$

$$\text{Ct} = -4*(10^{-15})\ *\text{RPM}^3 + 3*(10^{-10})\ *\text{RPM}^2 - 4*(10^{-6})\ \text{RPM} + 0.1983 \tag{17}$$

$$\text{Torque} = -4(10^{-14})\ \text{RPM}^3 + 4(10^{-9})\ \text{RPM}^2 - 7(10^{-5})\ *\text{RPM} + 0.3534 \tag{18}$$

3.4. ***Simulation***

Equations (11) – (17) are used to model the quadcopter system in MATLAB and SIMULINK. Figure 3 shows the open loop Simulink model of the quadcopter where equations (15) – (17) are used in the motor block while equations (11) – (14) are split into two blocks as Linear and Rotational dynamics.

Additionally, a disturbance block is also created in the simulation. The disturbance here is assumed to be wind gust which will be acting in x, y and z axis. This functional block is coded in such a way that random values about a specific axis will be given as an input to that block to which the block will give an output which will be given as an input to the linear dynamics. Hence, this disturbance block is a major block and has a great impact on the dynamics of the quadcopter.

PID Controller is one of the sought-after controllers in control theory because of its simplicity. Since this paper deals with the actuators represented in the form of mathematical equations using physical parameters, it is assumed that all four actuators (motors) are identical in every aspect. This may not be practically identified, but since we are dealing with this assumption in this research, a basic PID controller itself holds good to perform the time domain analysis. Figure 4 shows the completed model with attitude and altitude controller being implemented. It is important to note that the yaw, roll and pitch rates are controlled and not their angles.

The results for both the open loop response and closed loop response of the quadcopter are discussed in the next section where time domain parameters will be analyzed in detail.

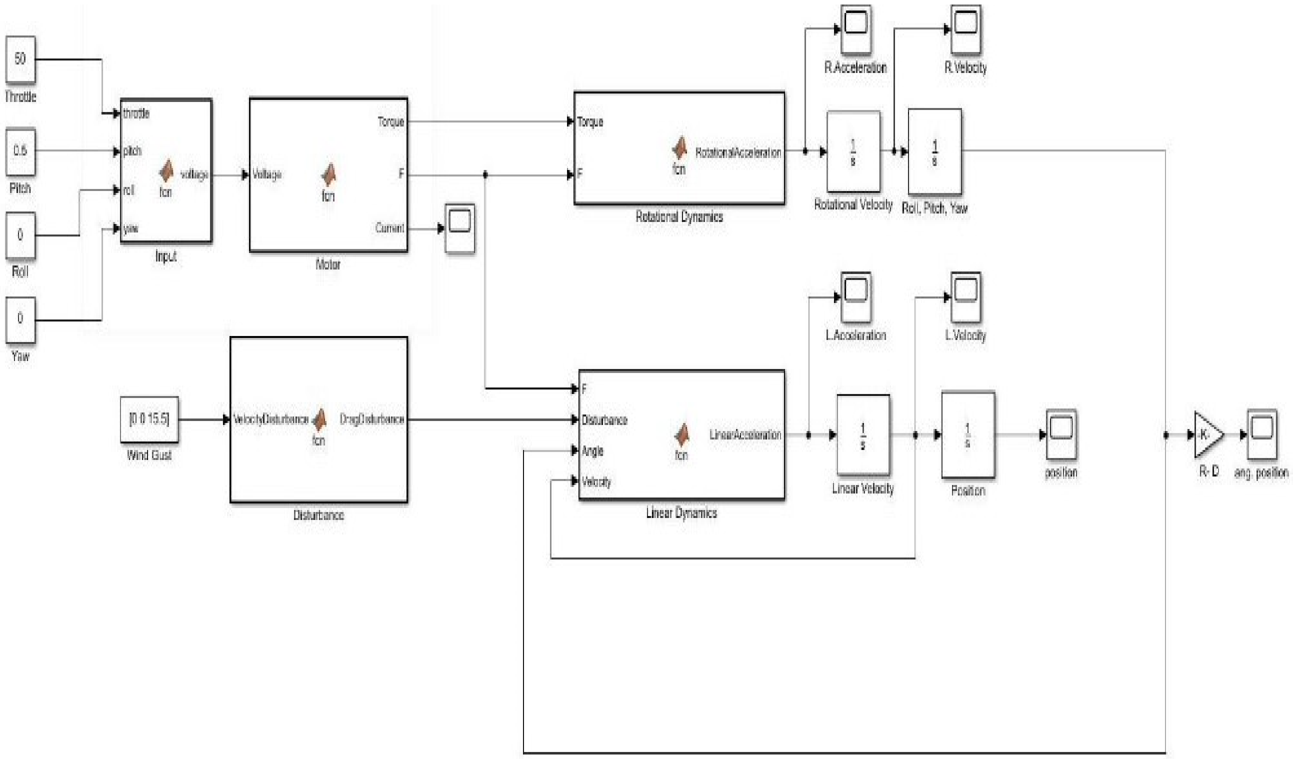

Figure 3. Open Loop Simulink Model of Quadcopter.

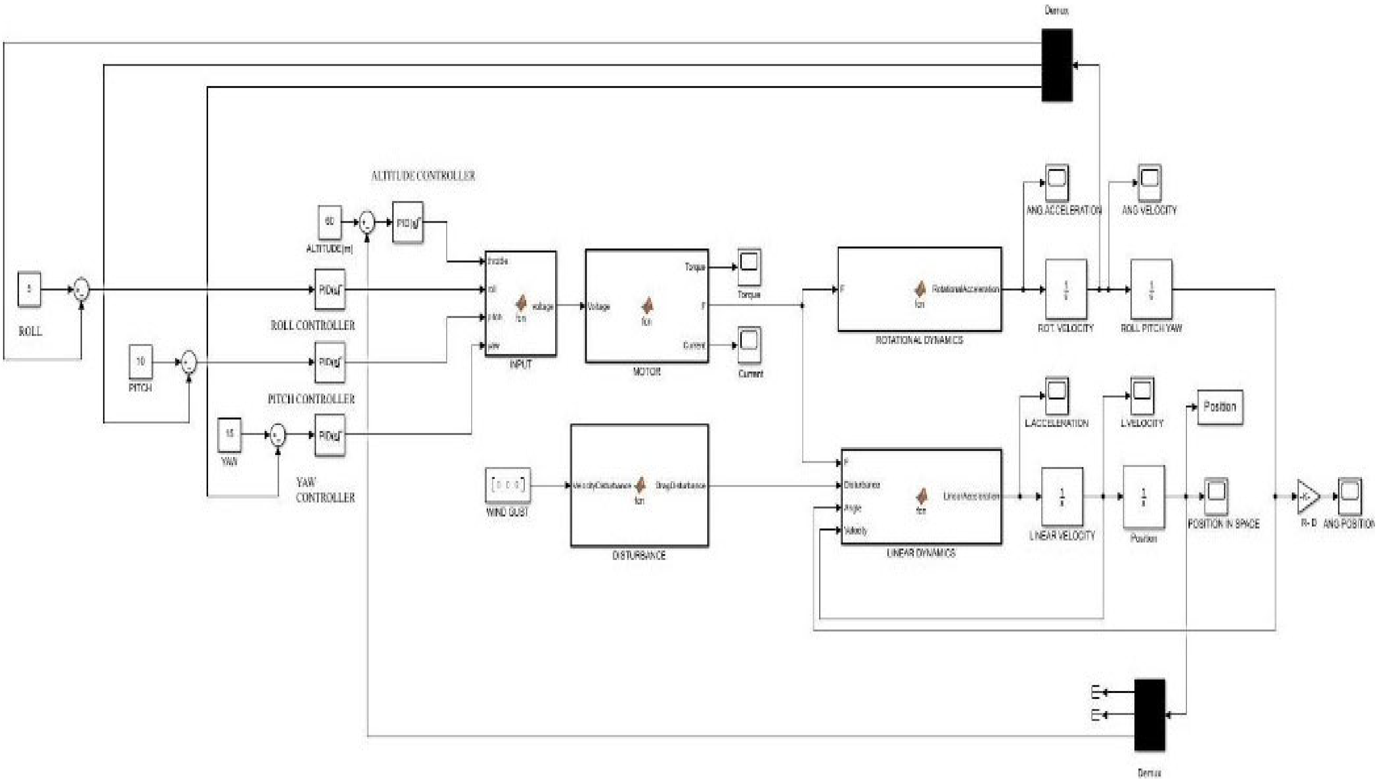

Figure 4. Model with attitude and altitude controller.

4. Results

This section shows the simulated response of the modeled quadcopter system. The first part includes the open loop response for various inputs like pitch input to check the functionality of the system. The second part includes the altitude and attitude control for the modeled system. The controller gain values for which optimal result is obtained will be discussed and the time domain parameters.

4.1. *Open Loop Response*

To check the functionality of the system, a disturbance along z – axis alone is given and simulated to see the response. Disturbance is given in such a way that the wind gust is acting along the z-axis pointing towards the sky. It was observed that the drone has moved up to a certain height because of the wind gust, while the position in x – axis and y – axis is zero which hints that modeled system is working properly.

When a positive pitch is given the drone has to move forward (i.e.) the drone has to move forward in y – axis and should move backward when the pitch value is negative. The result shown in Figure 5 is obtained when a positive pitch value is given. Figure 5 shows the position of the quadcopter in 3-dimensional space and the y axis is positive which indicates that the quadcopter is moving forward.

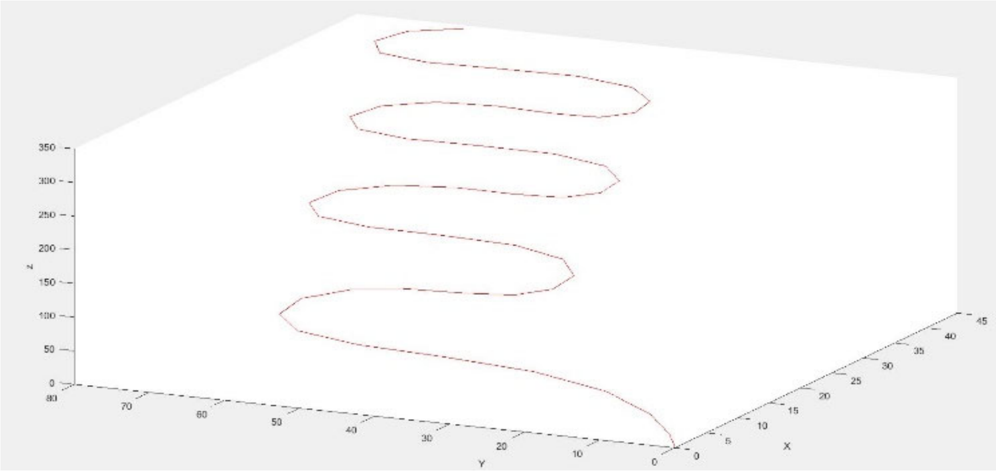

Figure 5. Position for Pitch Input

4.2. *Altitude and Attitude Control*

Separate PID controllers were designed for controlling the altitude, roll, pitch and yaw rates of the quadcopter. First the altitude controller alone is designed followed by attitude control.

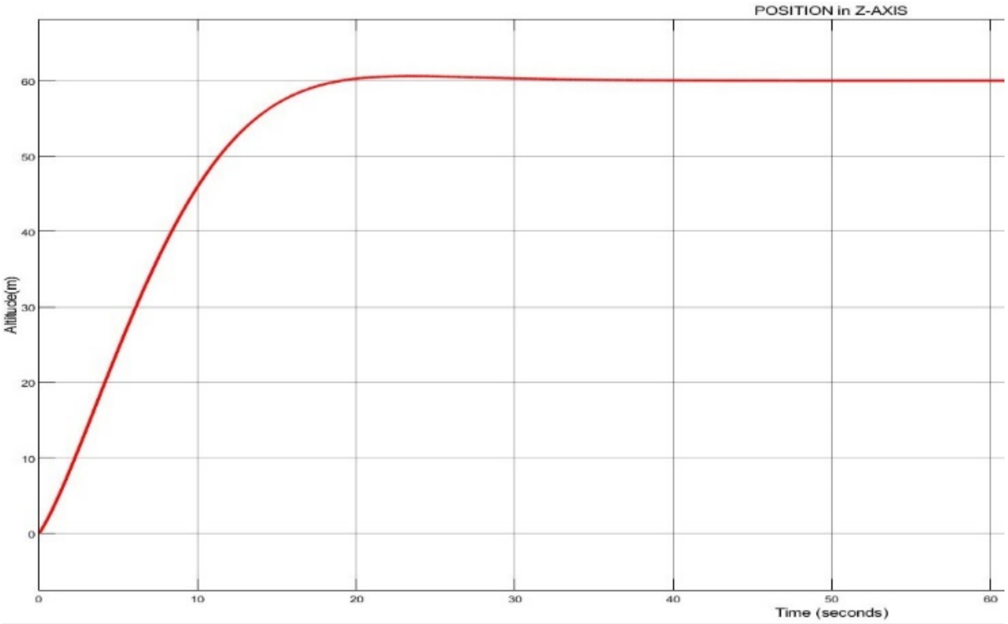

Figure 6. Altitude control result when altitude input is 60m

In simulation environment de-multiplexer block is used to take the value in z-axis alone which needs to be feed back to the designed PID controller. Here de-multiplexer does the job of the barometric pressure sensor which will be integrated in the quadcopter for measuring the altitude. Figure 6 shows the result of implementing altitude controller where the x-axis is time and the y-axis is altitude in meter. There is no steady state error and very minimal overshoot. The time domain specifications will be discussed along with time domain specification of attitude control.

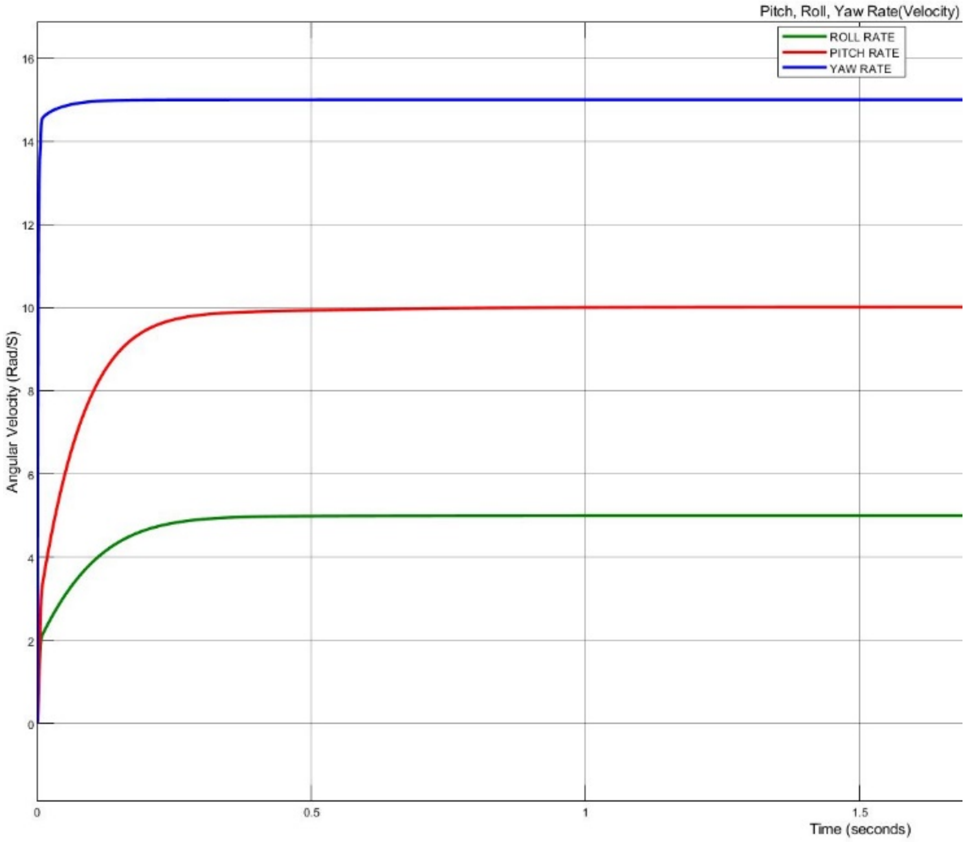

Figure 7. Response when Roll = 5 rad/s, Pitch = 10 rad/s and yaw = 15 rad/s.

Figure 7 shows the response after implementing PID controllers for roll, pitch and yaw rates where x-axis is time and the y-axis is the angular velocity. The response time is very low in the designed controller. Here another de-multiplexer is used to get the rotation rates of 3 axes separately which will be feedback for various controllers. In real-time, gyroscope will be integrated with the quadcopter which will give the roll, pitch and yaw rates to the controller based on which control actions will be taken.

4.3. *Controller Gains and Time Domain Parameters*

Table 1 shows the proportional integral and derivative gain values for various controllers. The response of pitch rate is very aggressive because of which peak overshoot occurs when the proportional gain is 0.3 which is adjusted to 0.2 to reduce the overshoot.

Table 1. Controller Gains.

	K_p	K_I	K_D
Altitude	0.31	0.035	0.9408
Roll Rate	1	0.0057	0.0071
Pitch Rate	0.2	0.0057	0.0071
Yaw Rate	1	0.0057	0.0071

The gain values for the proportional integral and derivative gains were based on trial-and-error method. Based on the studied research paper where similar parameters were considered for mathematical modelling, a range of values for each gain has been chosen. Then the values have been further narrowed down by trial and error.

Table 2. Time Domain Parameters.

Controller	Delay Time	Rise Time	Peak Time
Altitude	6.09 S	19.04 S	24.01 S
Roll	0.018 S	0.4 S	0.48 S
Pitch	0.026 S	0.9 S	0.99 S
Yaw	0.0001 S	0.13 S	0.2 S

Table 2 shows the time domain parameters for the controllers designed. From all the four controllers we can infer that the time difference between the rise time and peak time is very minimal which shows that there is very less amount of peak overshoot. Roll pitch and yaw controller provides a very aggressive response because of the which the rise time very low.

5. Conclusion

Considering the previous work and results found in this work it is easy to conclude that the quadcopter was controlled with four controllers and provides satisfactory results. In roll, pitch angle controller was designed where it took 3 seconds to compensate whereas in this research paper the maximum peak time itself is only 0.9 seconds which is 30% less than the existing systems. Considering altitude controller, research papers has designed controllers with peak time in milli seconds with respect to altitude, but yaw angle was neglected. Many research papers designed controllers considering roll and pitch angles. Yaw angle is neglected predominantly whereas in this research paper all three angle rates were considered along with altitude and respective PID controller were designed. Proceeding further, the gain values were decided using trial and error method in this research which could optimized in a better way using various other scientific techniques that might lead to better results. Currently, apart from quadcopters, the development of hybrid quadcopters and robotic systems that can operate in both aerial and terrestrial mode have increased. This concept is addressed but it has focused more on the mechanical design rather than the controller. The development of these kinds of systems will be a steppingstone towards flying cars. Hence, there is a need to develop controllers to control the system in both aerial and terrestrial mode.

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