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A COMPARATIVE ANALYSIS ON SYNCHRONIZATION OF TWO GENERAL CLASS OF CHAOTIC SYSTEMS USING NON-LINEAR FEEDBACK CONTROL AND FEEDBACK LINEARIZATION TECHNIQUES

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In this manuscript, two controlling techniques (Non-Linear Feedback control and Feedback Linearization) for synchronization of chaotic systems are proposed to attain synchronization between two generalized class of chaotic systems in master-slave configuration. An extensive analysis with systematic comparison between these two techniques are presented in this paper. Time delay is an inherent feature of a physical system, so study of the synchronization problem with time-delayed chaotic systems is the need of the hour. A detailed analysis is proposed for the same. Theoretical results are validated using MATLAB simulations.

1. Introduction

Since last three decades, chaos theory and its synchronization have been one of the most interesting topics due to its wide applications in different fields such as bio-medical engineering [24], chemical engineering [25], secure communications [26], etc. Dynamic behaviour of chaotic systems resembles that of a random system, yet followed by precise mathematical differential equations. The dynamical behaviour of these chaotic systems is highly dependent on the initial conditions, therefore even an infinitesimal change in these conditions cause drastic changes in the behaviour of its output. Because of these features, controlling the chaotic system is a very significant and demanding problem for the research fraternity. There exist many methods in literature about synchronization techniques for chaotic systems, such as nonlinear feedback control [1]- [3], feedback linearization technique [4]- [6], back stepping method [7], active control technique [8] and sliding mode control approach [9], etc. In most of these synchronization problems, master-slave configuration is used [10]- [18].

Generalized proof for synchronization of two different fractional order (FO) systems (namely, FO Lorenz system and FO Chen system), which are chaotic in nature has been demonstrated in [1]. In [2], nonlinear control technique is used to synchronize a pair of four chaotic systems and their global and asymptotic stability is also shown. Likewise, in [3] two identical Chen system and two non-identical systems (Genesio and Rossler systems) with fractional order have been formulated using Non-linear control scheme. The proposed scheme in [4] is applicable for tracking, regulation as well as synchronization of chaotic systems. Feedback linearization approach is proposed in [5] which is suppressing the oscillations that are generated from the mode-coupling mechanism and are not desirable for various applications of nonlinear dynamical friction system. Whereas, hyper chaotic discrete Rossler system is used in [6] to demonstrate the synchronization using model-matching approach. In order to apply back stepping method in [7], a strict feedback form is achieved by transformation of transformed modified Lorenz-Stenflo system (TMLS) and synchronization of two TMLS chaotic system with uncertain parameter is designed. Active control method is used in [8] for synchronization of similar as well as different fractional order chaotic systems. A 4D Rossler prototype-4 has been considered for controlling chaos by using sliding mode control and comparison is done by firstly using three controllers and then using two controllers in [9]. From [10]- [18], master-slave configuration is considered for controlling and synchronization of chaotic systems. In [19], an observer based (with time delay) fuzzy stochastic system for chaos synchronization along with asymptotic mean square stability is analyzed. Likewise, in [20], Nonlinear synchronization scheme is presented for the time delay chaotic systems which is also having channel noise, an integrator is used for suppressing the effect of channel noise. Using complex modified projection synchronisation (CMPS), a time delay complicated Chen chaotic system with parameter change and time delay factors is investigated in [21], and the strategy is implemented to a wireless body area network (WBAN) to encrypt and decode body data acquired by sensors.

Main objective of our study is to analyse some eye-catching concepts about synchronization approaches and provide a very systematic comparison between some of the techniques. The synchronization of two generalized identical chaotic systems with and without time-delay is examined and simulated intending to assess the performance of the proposed control strategies. The simulation results demonstrate the usefulness of the proposed synchronization control approaches.

Nonlinear feedback control is a common method among all due to its various advantages like (a) simplicity in configuration and implementation (b) takes less time for synchronization (c) controllers are economical, etc. The feedback linearization approach, on the other hand, is utilised when the system has to be linearized so that linear control methods may be implemented. The fundamental concept behind this approach is to employ state feedback and nonlinear transformation to convert a nonlinear system into a completely or partially linear system so that linear control methods may be used. The paper [22] aims to develop a control scheme using Feedback Linearization method to increase the life-span operations of a satellite. A simple and robust scheme based on Non-Linear control method for synchronization of Chua's system along with its application in secure communication is given in [23]. Comparison of these two techniques are not so much available in previous works. So, in this study, a systematic comparison for the same with and without time delays has been proposed.

This paper is organized in different sections as follows: Section-1 covers the introduction and a brief literature survey of existing work on this research domain. In Section-2, problem formulation as well as proposed controllers for both (i.e. nonlinear feedback control method and feedback linearization method) are formulated. In Section-3, example for synchronization of chaotic systems using both the techniques are elaborated and the results for the same are given in Section-4. Results obtained are enhanced and time delays are introduced in different states of the master and slave systems. Time delays are implicit nature of in any practical systems and analysis taking time delays has played a significant role in the control design problem. Thus analysis for the same is given in Section-5. In Section-6, numerical simulations and results for chaotic system with time-delays are provided. Finally, a brief conclusion is discussed in Section-7.

2. Problems Formulation and Proposed Controllers

A simple structure for synchronization of a generalized chaotic system is considered in master-slave configuration. In this paper, two types of synchronization methods are described and compared i.e. nonlinear feedback control method and feedback linearization method.

Let us define problems formulation for both techniques:

2.1 *Non-linear Feedback Control Technique*

The description for the master chaotic system is given below:

$$\begin{aligned} \dot{x_i} &= x_{i+1}, \quad 1 \le i \le (\text{n-1}) \\ \dot{x_n} &= \text{f (X)} \end{aligned} \tag{1}$$

where, the state variables are represented by X = $[x_1, x_2, \ldots, x_n]^T$, f(X) contains linear and non-linear functions of the master system and n is the order of the system. Similarly, the controlled slave system can be described as:

$$\dot{y_i} = y_{i+1}, \quad 1 \le i \le (\text{n-1})$$

$$\dot{y_n} = f(Y) + u_n \tag{2}$$

where $Y = [y_1, y_2, \ldots, y_n]^T$ represent the state variables, $u_n = [u_1, u_2, \ldots, u_n]^T$ are the control inputs applied to the slave system and it contains linear and nonlinear terms which is defined by f (Y).

The synchronization of the drive and the response chaotic systems can be achieved when the error dynamics between them tends to zero as time approaches infinity. The synchronizing error can be written as:

$$e_i = y_i - x_i, \quad i = 1,2,\ldots, n$$

Now, by taking the time derivative, we get:

$$\begin{aligned} \dot{e_i} &= \dot{y_i} - \dot{x_i}, \\ \dot{e_i} &= e_{i+1} + u_i, \quad 1 \le i \le (n-1) \\ \dot{e_n} &= f(Y) - f(X) + u_n \\ \dot{e_n} &= g(e) + H(X,e) + u_n \end{aligned} \tag{3}$$

where, the residual term is H (X, e) and g(e) is the residual error function. g(e) = $c_1 e_1 + c_2 e_2 + \ldots + c_n e_n$ and $c_1, c_2, \ldots, c_n$ are the constant values. The challenge is to design a feedback control law U = $[u_1, u_2, \ldots, u_n]^T$ such that the slave system follows the behaviour of the master system.

Controller design using Non-Linear Feedback control technique:

The proposed structure of the controllers for this method is given as follows:

$$\begin{aligned} u_i &= -k_i e_i, \\ u_n &= -k_n e_n - H(X,e) \end{aligned} \tag{4}$$

where, K = diag (k_1, k_2, ..., k_n) is the (n×n) gain matrix. Now applying the controllers (4) on (3), we get:

$$\begin{aligned} \dot{e_1} &= e_2 - k_1 e_1, \\ \dot{e_2} &= e_3 - k_2 e_2, \\ &\vdots \\ \dot{e_n} &= c_1 e_1 + c_2 e_2 + \ldots + c_n e_n - k_n e_n \end{aligned}$$

Now, one can write the error dynamics in matrix form given below:

$$\begin{pmatrix} \dot{e_1} \\ \dot{e_2} \\ . \\ . \\ . \\ \dot{e_n} \end{pmatrix} = - \begin{pmatrix} k_1 & -1 & \ldots & 0 \\ 0 & k_2 & \ldots & 0 \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ -c_1 & -c_2 & \ldots & -(c_n - k_n) \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ . \\ . \\ . \\ e_n \end{pmatrix}$$

$$\dot{e} = -A\,e \tag{5}$$

Note: Now the problem is reduced to choose proper values of K gain matrix, such that the coefficient matrix A in error dynamics in (5) becomes Positive Definite Matrix (PDM). Thus, it can be seen that the error dynamics of master and slave systems is converging asymptotically, hence the synchronization is achieved.

2.2 *Feedback Linearization Technique*

The generalized structure of the master chaotic system can be given as:

$$\begin{aligned} \dot{x_i} &= x_{i+1}, \quad 1 \le i \le (n-1) \\ \dot{x_n} &= G(X) + u_m \\ z &= h(X) \end{aligned} \qquad (6)$$

where, the state variables are denoted by X = $[x_1, x_2, \dots, x_n]^T$, n is the order of the system, G(X) contains linear and non-linear functions of the master system, u_m is the master input and z is the output of the master system.

Likewise, the slave system can be written as:

$$\begin{aligned} \dot{y_i} &= y_{i+1}, \quad 1 \le i \le (n-1) \\ \dot{y_n} &= G(Y) + u_s \\ w &= p(Y) \end{aligned} \qquad (7)$$

where Y = $[y_1, y_2, \dots, y_n]^T$ are the state variables, u_s is the slave controller, n is the order of the system and G(Y) contains linear and non-linear functions of the system and w is the output of this system(slave).

The matrix form of the master system (6) can be written as:

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ . \\ . \\ . \\ \dot{x_{n-1}} \\ \dot{x_n} \end{pmatrix} = \begin{pmatrix} x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \\ G(X) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 1 \end{pmatrix} U_m$$

$$\dot{X} = A(X) + B(X)\ U_m \qquad (8)$$

where, $\dot{X} = [\dot{x_1}, \dot{x_2}, \dots \dot{x_{n-1}}, \dot{x_n}]^T$, A(X) = $[x_2, x_3, \dots, x_n, G(X)]^T$ and B(X) = $[0,0,\dots,1]^T$.

Definition 1: (Lie Derivative) The feedback linearization method aims to produce a transformed system whose states are the output and the first (n-1) derivative, this can be achieved by using Lie Derivative.

From (6), we know that: z = h(X)

$$\dot{z} = \frac{dh(X)}{dt} = \frac{dh(X)}{dX} . \dot{X}$$

Putting the value of ˙X from (8), we get:

$$\dot{z} = \frac{dh(X)}{dX} .\mathrm{A(X)} + \frac{dh(X)}{dX} .\mathrm{B(X)}\ U_m$$

The Lie derivative of h(X) along A(X) and B(X) can be defined as follows:

$$\frac{dh(X)}{dX} .\mathrm{A(X)} = L_A\mathrm{h(X)}$$

$$\frac{dh(X)}{dX} .\mathrm{B(X)} = L_B\mathrm{h(X)}$$

Therefore: $\dot{z} = L_A\mathrm{h(X)} + L_B\mathrm{h(X)}\ U_m$

Likewise, we can also calculate the Lie derivative for slave system which can be written as:

$$\dot{w} = L_A\mathrm{p(Y)} + L_B\mathrm{p(Y)}\ U_s$$

Definition 2: (Relative Degree) This can be termed as the number of times we differentiate the output with respect to time so that the input explicitly appears in the output equation. Let us denote relative degree of master system as r_m and the relative degree of slave system as r_s.

For instance, if we consider h(X) $= x_1$ and p (Y) = y_1 for the analysis. By considering this assumptions, we can show that the master as well as the slave systems will possess relative degree of n (i.e., $r_m = r_s$ = n) that is equivalent to the order of the system. Hence, both the master and slave systems are fully linearizable.

Controller using Feedback Linearization method

A new coordinate transformation is needed in this method. Let us consider ζ as a new auxiliary variable of the transformed coordinate system such that:

$$\zeta = [\zeta_1, \zeta_2, \dots, \zeta_n]^T$$

The values will be given as:

$$\zeta_1 = e_1 = \mathrm{p(Y)} - \mathrm{h(X)},$$

$$\begin{cases} \dot{\zeta_i} = \zeta_{i+1}, & i = 1,2,\ldots,(n-1) \\ \dot{\zeta_n} = \dot{y_n} - \dot{x_n} & = \mathrm{G(Y)} + u_s - \mathrm{G(X)} - u_m \end{cases} \tag{9}$$

We assume u_m as disturbance or unwanted input present in the master system (6) and we want to decouple it from the new transfer coordinate system (9) by incorporating appropriate control law, therefore u_m = 0, then

$$\dot{\zeta_n} = \mathrm{G(Y)} + u_s - \mathrm{G(X)} \tag{10}$$

Now the controller is designed as:

$$u_s = -G(Y) + G(X) + v \quad (11)$$

From (11) and (10), we get:

$$\dot{\zeta}_n = v$$

Let us consider $v = -D_0\,\zeta_1 - D_1\,\zeta_2 \ldots - D_{n-1}\zeta_n$, then

$$\dot{\zeta}_n = -D_0\,\zeta_1 - D_1\,\zeta_2 \ldots - D_{n-1}\zeta_n$$

where $D_0, D_1, \ldots, D_{n-1}$ are the constant real coefficients.

Hence, the new coordinate system can be written in matrix form as:

$$\begin{bmatrix} \dot{\zeta}_1 \\ \dot{\zeta}_2 \\ . \\ . \\ . \\ \dot{\zeta}_n \end{bmatrix} = \begin{pmatrix} 0 & 1 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ -D_0 & -D_1 & \ldots & -D_{n-1} \end{pmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ . \\ . \\ . \\ \zeta_n \end{bmatrix} \quad (12)$$

By choosing the appropriate values for $D_0, D_1, \ldots, D_{n-1}$, we can ensure the poles of the new transformed system (12) lie on Left Hand Side (LHS) of S-plane which in return ensures, the error dynamics will also be stable (converging) in nature.

3. Example: Arneodo-Arneodo Synchronization

In order to illustrate the theoretical results discussed in section-2, let us consider a system that falls under the generalized structure mentioned in section-2 (1), which is Arneodo (master) System. The mathematical description of Arneodo system is:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -bx_2 - x_1^2 + a\,x_1 - x_3 \end{aligned} \quad (13)$$

here, a > 0 and b > 0 which are the system parameters.

3.1 *Controller Design using Non-Linear Feedback Control Method*

The controlled slave system can be defined as:

$$\begin{aligned} \dot{y}_1 &= y_2 + u_1, \\ \dot{y}_2 &= y_3 + u_2, \\ \dot{y}_3 &= -by_2 - y_1^2 + a\,y_1 - y_3 + u_3 \end{aligned} \quad (14)$$

Now we can develop the controllers based on (4):

$$u_1 = -k_1 e_1$$

$$u_2 = -\ k_2 e_2 \tag{15}$$
$$u_3 = -\ k_3 e_3 + e_1^2 + 2x_1 e_1$$

From (5), we can write the matrix form error dynamics as given below:

$$\begin{bmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \end{bmatrix} = -\begin{bmatrix} k_1 & (-1) & \dots & 0 \\ 0 & k_2 & \dots & (-1) \\ (-a) & b & \dots & (1+k_3) \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Now, choosing proper values of k_1, k_2 and k_3 we can ensure the Coefficient Matrix (CM) of the error dynamics to be Positive Definite Matrix (PDM), so as to conclude that the error dynamics represented by the system $\dot{e} = -Ae$ is asymptotically converging in nature, hence stable.

To be a PDM, the CM has to have all the n Leading Principal Minors (LPM) to be positive. Here, n = 3 in our case. The LPM of order 1 is k_1; which is to be > 0.
And correspondingly the LPM of order 2 is $\begin{bmatrix} k_1 & (-1) \\ 0 & k_2 \end{bmatrix}$; whose determinant to be > 0 which is $(k_1 k_2) > 0$.

Finally, LPM of order 3 is the CM itself and it has to follow the condition det(CM)>0; i.e. $(k_2 + k_2 k_3 + b) > 0$.

3.2 *Designing Controller using Feedback Linearization Method*

Consider the master system as given below:

$$\begin{aligned} \dot{x_1} &= x_2, \\ \dot{x_2} &= x_3, \\ \dot{x_3} &= -\ b x_2 - x_1^2 + a\ x_1 - x_3 + u_m \\ z &= h(X) = x_1 \end{aligned} \tag{16}$$

where, a > 0 and b > 0 which are the system parameters.

Correspondingly, the slave system can be considered as:

$$\begin{aligned} \dot{y_1} &= y_2 \\ \dot{y_2} &= y_3 \\ \dot{y_3} &= -\ b y_2 - y_1^2 + a\ y_1 - y_3 + u_s \\ w &= p(Y) = y_1 \end{aligned} \tag{17}$$

where, a > 0 and b > 0 which are the system parameters.

By following (11), we can formulate the controller as follows:

$$u_s = -\ a\ (y_1 - x_1) + b(y_2 - x_2) + y_3 - x_3 + y_1^2 - x_1^2 + v \tag{18}$$

where;

$$v = -D_0\ \zeta_1 - D_1\ \zeta_2 - D_2 \zeta_3,$$

Choosing the poles to be on LHS of the S-plane, we consider $[D_0, D_1, D_2] = [125, 75, 15]$, which ensures that the auxiliary system is stable, subsequently error dynamics will also be stable (converging) in nature.

4. Numerical Simulations and Results

For the simulations, MATLAB R2013a software is used. For solving Ordinary Differential Equations, ode45 solver is used.

Note: The controllers are activated at t = 20 seconds for both the synchronizing methods.

Non-Linear Feedback Control method

The parameter values are taken as a =15 and b =10. The gains are chosen as $k_1 = k_2 = k_3$ = 15. Time step for solving the ODEs are taken as 0.01 second and initial values are considered to be X (0) = [1,0.5,2] and Y (0) = [0.7,0.9,0.8].

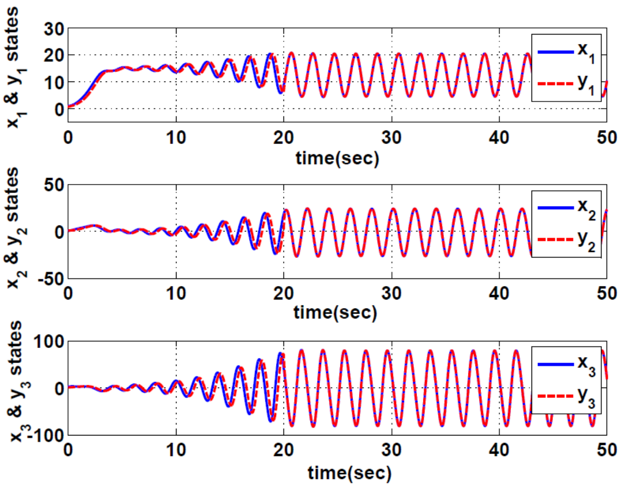

Figure 1. Trajectories of synchronized states of master system and slave system vs time plot (using Non-Linear Feedback Control method)

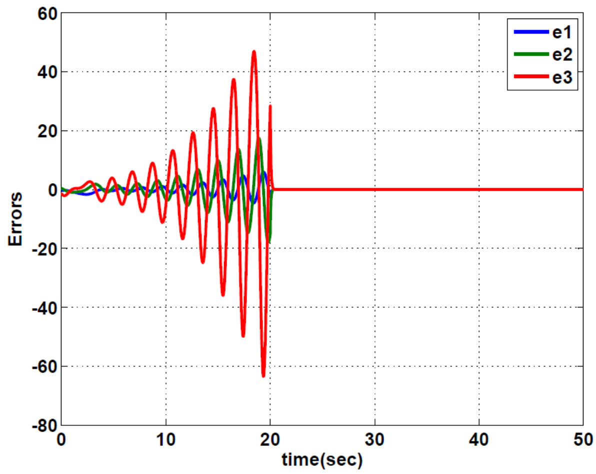

Figure 2. Trajectories of synchronized error dynamics (using Non-Linear Feedback Control method)

In Figure 1, we can see, the states of slave system are overlapping on the states of the master system ($y_1 \rightarrow x_1, y_2 \rightarrow x_2, y_3 \rightarrow x_3$) and in Figure 2, error e_1, e_2, e_3 are converging to zero in finite duration. Therefore, one can realize that, the systems (master and slave) are synchronized.

Feedback Linearization method

The system parameter values are taken as a =15 and b =10. To complete the simulation of the ODEs, initial conditions are taken as X (0) = [1,0.5,2] and Y (0) = [0.7,0.9,0.8]. The time step is taken as 0.01 second.

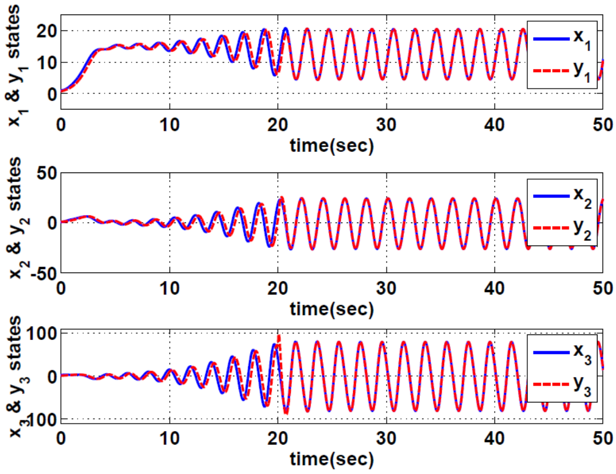

Figure 3. Trajectories of synchronized states of master and slave systems vs time plot (using Feedback Lineariza-tion method)

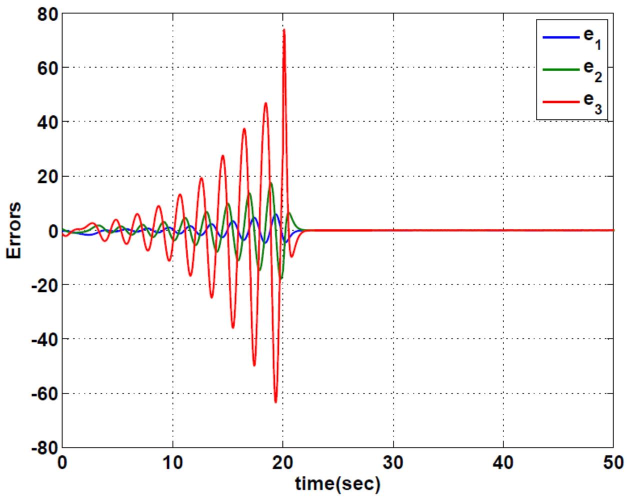

Figure 4. Trajectories of synchronized error dynamics (using Feedback Linearization method)

In Figure 3 it is seen, the slave system trajectories are following the trajectories of master system and Figure 4 depicts that the error dynamics are converging to zero as time is increase-ing. Therefore, we can state that the master system and the slave system are synchronized using feedback linearization method.

5. An extended case: Study of the same synchronization problem with Time-Delayed Chaotic systems

Time-delays are present almost in every physical systems and the effect of time-delay on dynamics of the system is inescapable. Time-delay systems can exhibit multi-stability and can produce even more complex dynamics which can help in enhancing the security of transmitted data. In this section, time-delays have been introduced on different states of Arneodo system and the synchronization is achieved using both the techniques.

5.1 *Non-linear Feedback Control method*

The Arneodo (master) and controlled Arneodo (slave) systems with time-delays can be written as:

$$\dot{x_1}(t) = x_2(t)$$
$$\dot{x_2}(t) = x_3(t)$$
$$\dot{x_3}(t) = -bx_2(t-\tau_2) - x_1^2(t) + a\,x_1(t-\tau_1) - x_3(t-\tau_3)$$

&

$$\dot{y_1}(t) = y_2(t) + u_1$$
$$\dot{y_2}(t) = y_3(t) + u_2$$
$$\dot{y_3}(t) = -by_2(t-\tau_5) - y_1^2(t) + a\,y_1(t-\tau_4) - y_3(t-\tau_6) + u_3$$

Using the procedure described in section-2, the controllers for the same can be given as follows:

$$u_1 = -k_1 e_1$$
$$u_2 = -k_2 e_2$$
$$u_3 = -k_3 e_3 - a[y_1(t-\tau_4) - x_1(t-\tau_1)] + b[y_2(t-\tau_5) - x_2(t-\tau_2)] + [y_3(t-\tau_6) - x_3(t-\tau_3)] + y_1^2(t) - x_1^2(t)$$

5.2. *Feedback Linearization technique*

The Arneodo (master) and controlled Arneodo (slave) systems with time-delays can be written as:

$$\dot{x_1}(t) = x_2(t)$$
$$\dot{x_2}(t) = x_3(t)$$
$$\dot{x_3}(t) = -bx_2(t-\tau_2) - x_1^2(t) + a\,x_1(t-\tau_1) - x_3(t-\tau_3) + u_m$$
$$z = x_1(t)$$

&

$$\dot{y_1}(t) = y_2(t)$$
$$\dot{y_2}(t) = y_3(t)$$
$$\dot{y_3}(t) = -by_2(t-\tau_5) - y_1^2(t) + a\,y_1(t-\tau_4) - y_3(t-\tau_6) + u_s$$
$$w = y_1(t)$$

The controller using Feedback Linearization Technique can be derived as:
Let us take the auxiliary variable $\zeta_i = y_i(t) - x_i(t)$.

Now, taking time derivative of the auxiliary variable we get:

$$\dot{\zeta}_1 = \zeta_2 = \dot{y}_1(t) - \dot{x}_1(t) = y_2(t) - x_2(t)$$
$$\dot{\zeta}_2 = \zeta_3 = \dot{y}_2(t) - \dot{x}_2(t) = y_3(t) - x_3(t)$$
$$\dot{\zeta}_3 = \dot{y}_3(t) - \dot{x}_3(t) \tag{19}$$
$$\dot{\zeta}_3 = ay_1(t-\tau_4) - by_2(t-\tau_5) - y_3(t-\tau_6) - y_1^2(t) + u_s$$
$$-[ax_1(t-\tau_1) - bx_2(t-\tau_2) - x_3(t-\tau_3) - x_1^2(t) + u_m]$$

Now, we know that $u_m = 0$. From (10) we get:

$$\dot{\zeta}_3 = ay_1(t-\tau_4) - by_2(t-\tau_5) - y_3(t-\tau_6) - y_1^2(t) + u_s$$
$$-[ax_1(t-\tau_1) - bx_2(t-\tau_2) - x_3(t-\tau_3) - x_1^2(t)]$$

By taking:

$$u_s = -ay_1(t-\tau_4) + by_2(t-\tau_5) + y_3(t-\tau_6) + y_1^2(t)$$
$$+ax_1(t-\tau_1) - bx_2(t-\tau_2) - x_3(t-\tau_3) - x_1^2(t) + \mathrm{v} \tag{20}$$

and considering $\mathrm{v} = -P_0\zeta_1 - P_1\zeta_2 - P_2\zeta_3$ and applying the controller from (20) in (19), we get:

$$\begin{pmatrix} \dot{\zeta}_1 \\ \dot{\zeta}_2 \\ \dot{\zeta}_n \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -P_0 & -P_1 & -P_2 \end{pmatrix} \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{pmatrix} \tag{21}$$

Choosing $[P_0, P_1, P_2] = [27,27,27]$, we ensure that system (21) has its poles in LHS of the S-plane, hence the auxiliary system is proved to be stable, subsequently error dynamics will also be stable (converging) in nature.

6. Numerical Simulations and Results for Time-delay Arneodo system

For the simulations, MATLAB R2013a software is used.
For solving Delay Differential Equations, dde23 solver is used.

Note: At t = 20 seconds, the controllers are activated for both the synchronizing methods.

Non-Linear Feedback control method

The system parameter values are taken as a =15 and b =12. The gains are chosen as $k_1 = k_2 = k_3 = 5$. The values of Time-Delays for our analysis are taken as: $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = 0.002$ second. The initial values to complete the simulation are taken as X (0) = [2, -1,3] and Y (0) = [4,1,-5]. Time step is considered as 0.01 second.

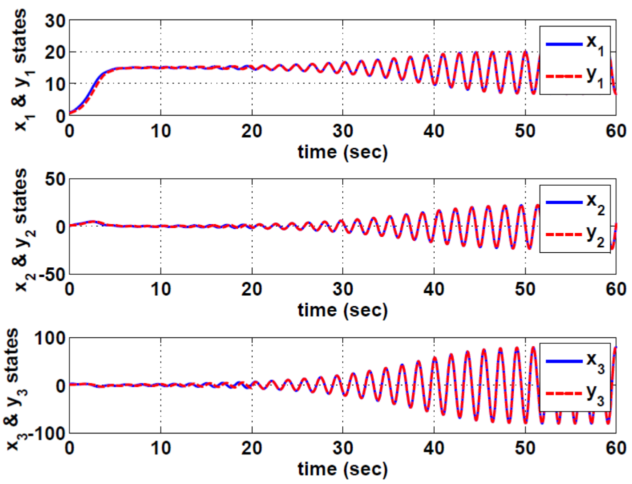

Figure 5. Trajectories of synchronized states of the master and slave systems vs time after applying time-delay

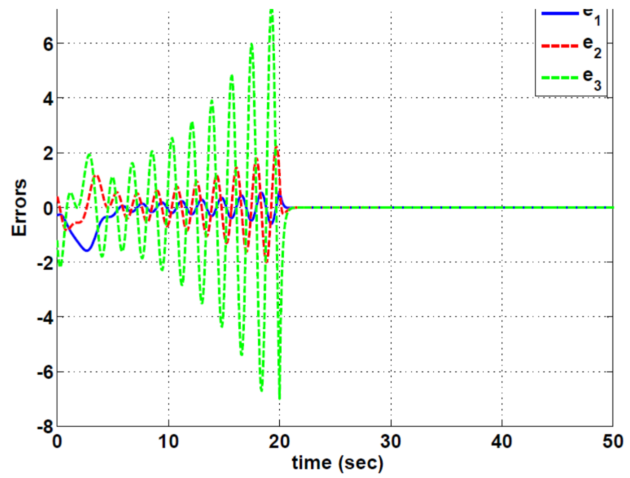

Figure 6. Trajectories of synchronized error dynamics after applying time-delay (using Nonlinear feedback method)

Figure 5 depicts, the trajectories of time delayed slave system are following the trajectories of the time delayed master system. From Figure 6, we can conclude that errors are approaching to zero as time is increasing which means the master system and the slave system are synchronized.

Feedback Linearization method

The parameter values for the analysis are taken as a =15 and b =12. The gains are chosen as P_0= P_1= P_2= 27. The values of Time-Delays for our analysis are taken as to be $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6$ = 0.002 second. The initial values to complete the simulation are taken as X (0) = [2, -1,3] and Y (0) = [4,1,-5]. Time step is considered as 0.01 second.

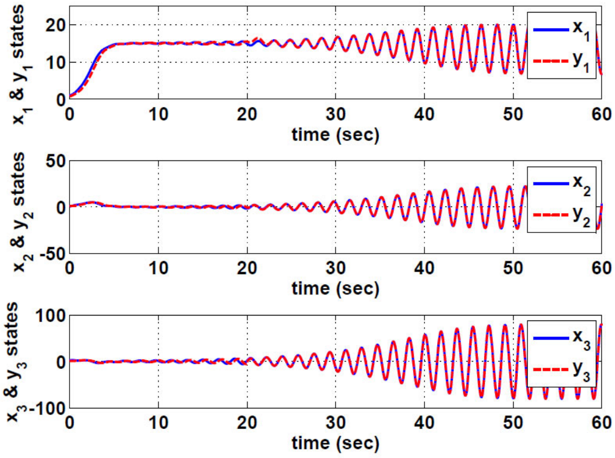

Figure 7. Trajectories of synchronized states of master system and slave system vs time after applying time- delay (using Feedback Linearization method)

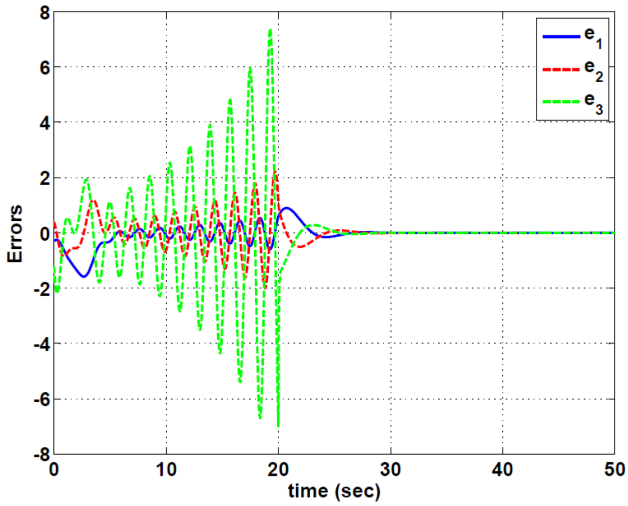

Figure 8. Trajectories of synchronized error dynamics vs time after applying time-delay (using Feedback Linearization method)

In Figure 7 it is seen, the trajectories of time delayed slave system are following the trajectories of the time delayed master system ($y_1 \to x_1, y_2 \to x_2, y_3 \to x_3$) and Figure 8 depicts the error dynamics are converging to zero as time is increasing. Therefore, we can declare the time delayed master system and the time delayed slave system are synchronized using feedback linearization method.

7. Conclusion

A constructive analysis for Non-Linear Feedback Control and Feedback Linearization techniques are discussed in this paper. Systematic step by step procedures are carried out throughout this work. A sound comparison between Time-Delay synchronization and without Time-Delay synchronization is presented with extensive simulation results.

A generalized class of chaotic system is considered for our analysis and all the steps to develop the controllers are presented in a systematic manner. All these controllers hold good for the systems; such as: Genesio-Tesi system, Jerk system, Hyper-jerk system and Duffing Oscillator etc.

It is seen that in non-linear control technique there is a requirement of n controllers to achieve the synchronization of master system and slave system, where n is order of the system. Whereas, in feedback linearization approach only 1 controller is sufficient to achieve the same task. This is true for both Time-Delayed and without Time-Delayed chaotic systems.

The controllers in non-linear feedback control are more complex for viable usage. Comparing (15) and (13), one can promptly observe that the controller is more complex than the system itself. The complexity of controllers is a critical issue in actual implementation, such as in electronics and engineering applications.

In this paper, a specific generalized structure is considered for validating the theoretical and simulation results. One can study the same with different chaotic systems. The same study can also be extended with the incorporation of external disturbance and model-uncertainties to the systems.

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